Hong Kong Mathematics Olympiad (2002 – 2003)
Heat Event (Group)
香港数学竞赛 (2002 – 2003)
初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 若 $k=\frac{1}{4\times5\times6}+\frac{1}{5\times6\times7}+\frac{1}{6\times7\times8}+\cdots+\frac{1}{99\times100\times101}$，求 k 的值。

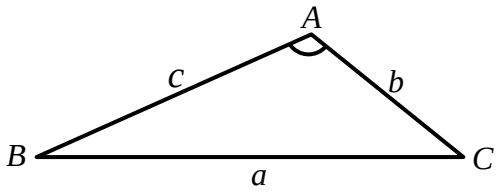
If $k=\frac{1}{4\times5\times6}+\frac{1}{5\times6\times7}+\frac{1}{6\times7\times8}+\cdots+\frac{1}{99\times100\times101}$, find the value of k.

2. 若 $x>1$， $y>0$，且 $x^y+x^{-y}=2\sqrt{2}$ 及 $x^y-x^{-y}=k$，求 k 的值。

Suppose $x^y+x^{-y}=2\sqrt{2}$ and $x^y-x^{-y}=k$, where $x>1$ and $y>0$, find the value of k.

3. 图一中， $\angle A:\angle B:\angle C=3:2:1$， $a:b:c=2:k:1$，求 k 的值。

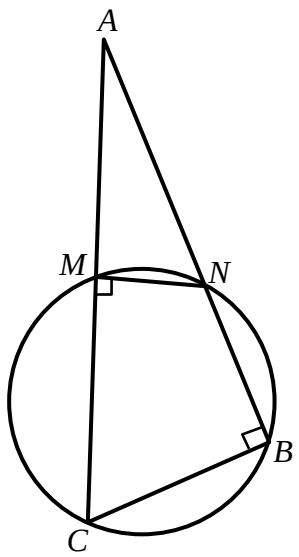
In Figure 1, $\angle A:\angle B:\angle C=3:2:1$, $a:b:c=2:k:1$, find the value of k.



图一
Figure 1

4. 图二中，AMC 和 ANB 为直线，$\angle NMC = \angle NBC = 90°$，$AB = 4$，$BC = 3$，$\Delta AMN$ 及 ΔABC 面积之比为 $1:4$。求圆形 $BNMC$ 的半径。

In Figure 2, AMC and ANB are straight lines, $\angle NMC = \angle NBC = 90°$, $AB = 4$, $BC = 3$, areas of ΔAMN and ΔABC are in the ratio $1:4$. Find the radius of circle $BNMC$.



图二

Figure 2

5. 对任意实数 a，方程 $x^2 + ax + 3b - a + 2 = 0$ 都有实数解，求 b 的最大值。

If the equation $x^2 + ax + 3b - a + 2 = 0$ has real root(s) for any real number a, find the maximum value of b.

6. 设抛物线 $y = 4x^2 - 5x + c$ 与 x-轴相交于 $(\cos\theta, 0)$ 及 $(\cos\phi, 0)$。若 θ 和 ϕ 分别为一直角三角形中两锐角的角度，求 c 的值。

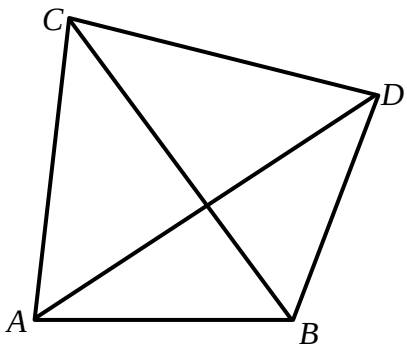
Suppose the parabola $y = 4x^2 - 5x + c$ intersects the x-axis at $(\cos\theta, 0)$ and $(\cos\phi, 0)$ respectively. If θ and ϕ are two acute angles of a right –angled triangle, find the value of c.

7. 设直线 $y+3x-4=0$ 与抛物线 $y=x^2$ 相交于 A 及 B。若 O 为原点，求 ΔOAB 的面积。

Suppose the straight line $y+3x-4=0$ intersects the parabola $y=x^2$ at points A and B respectively. If O is the origin, find the area of ΔOAB.

8. 图三中，$AC=BC=CD$，$\angle ACB=80°$。若 $\angle ADB=x°$，求 x 的值。

In Figure 3, $AC=BC=CD$, $\angle ACB=80°$. If $\angle ADB=x°$, find the value of x.

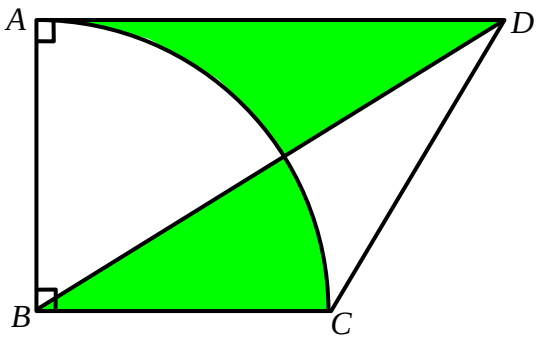


图三

Figure 3

9. 图四中，扇形 ABC 为半径是 4 cm 的圆的四分之一，且两个阴影部分的面积相等。设梯形 $ABCD$ 的面积为 A cm^2，求 A 的值。(取 $\pi=3.14$)

In Figure 4, the sector ABC is one quarter of a circle with radius 4 cm. Suppose the areas of the two shaded parts are equal. Let the area of the trapezium $ABCD$ be A cm^2, find the value of A. (Take $\pi=3.14$)

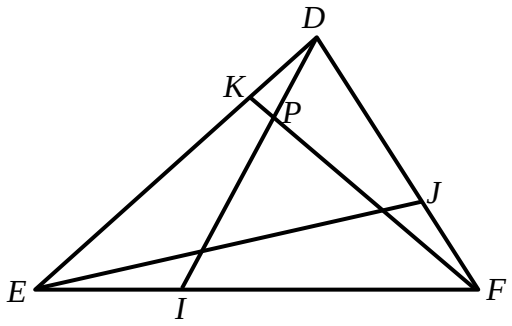


图四

Figure 4

10. 图五中，ΔDEF 的面积是 30 cm^2。EIF 、 DJF 及 DKE 皆为直线。P 是 DI 和 FK 的相交点，$EI : IF = 1 : 2$，$FJ : JD = 3 : 4$，$DK : KE = 2 : 3$。设 ΔDFP 的面积为 $B\text{ cm}^2$，求 B 的值。

In Figure 5 , the area of ΔDEF is 30 cm^2 . EIF , DJF and DKE are straight lines . P is the intersection point of DI and FK . Let $EI : IF = 1 : 2$, $FJ : JD = 3 : 4$, $DK : KE = 2 : 3$. Let the area of ΔDFP be $B\text{ cm}^2$, find the value of B .



图五

Figure 5